Horizontal focusing conditions for crystal monochromator

Define $2\theta$ as the scattering angle of the central part of the beam and $\phi$ as the angle between the incident beam and the monochromator as shown on the sketch below. $\rho$ denotes the displacement from the center of the monochromator in the direction indicated. Denote by $\psi(\rho)$ the rotation with respect to the monochromator frame of the MCU at position $\rho$. The positive sense of rotation for individual MCUS ($\psi(\rho)$) coincides with the positive sense of rotation for the entire device ($\phi$).

Monochromatic focusing at specified $2\theta$ is achieved for

$$\phi = A \tan \left( \frac{\sin 2\theta}{\cos 2\theta + L_i / L_0} \right) \quad \text{for} \quad \cos 2\theta + L_i / L_0 \geq 0 \quad \text{and} \quad \phi = A \tan \left( \frac{\sin 2\theta}{\cos 2\theta + L_i / L_0} \right) + \pi \quad \text{for} \quad \cos 2\theta + L_i / L_0 < 0$$

and

$$\psi(\rho) = A \tan \left( \frac{\sin 2\theta}{\cos 2\theta + L_i / L_0 - (\rho / L_0)\sqrt{1 + (L_i / L_0)^2 + 2(L_i / L_0)\cos 2\theta}} \right) - \theta$$

Point to point focusing at specified $2\theta$ and $\phi$ is achieved for

$$\psi(\rho) = \theta - \phi - \frac{1}{2} \left( A \tan \left( \frac{\rho \sin \phi}{L_0 - \rho \cos \phi} \right) + A \tan \left( \frac{\rho \sin (2\theta - \phi)}{L_i + \rho \cos (2\theta - \phi)} \right) \right)$$

This formula does not yield the same result as the one above when $\phi$ is chosen to achieve monochromatic point to point focusing. This is because we have optimized for monochromaticity in the former case and for spot size in the latter case.